Technical Comments

AIAA 82-4081

Comment on "Model-Following System with Assignable Error Dynamics and its Application to Aircraft"

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KAWAHATA considers in Ref. 1 the model following problem: given a completely controllable and observable linear time-invariant plant,

$$\begin{bmatrix} \dot{x} \\ y \end{bmatrix} = \begin{bmatrix} F & G \\ H & D \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} \tag{1}$$

and model,

$$\begin{bmatrix} \dot{x}_m \\ y_m \end{bmatrix} = \begin{bmatrix} F_m & G_m \\ H_m & D_m \end{bmatrix} \begin{bmatrix} x_m \\ u_m \end{bmatrix} \tag{2}$$

find the control u which causes y to track y_m for an unspecified u_m. The number of outputs, y, (an $l \times 1$ vector) is assumed to equal the number of controls, u, (an $m \times 1$ vector). Kawahata required that D and D_m in Eqs. (1) and (2) be zero and restricted the control u to have the following form

$$u = K_x x + K_{xm} x_m + K_{um} u_m \tag{3}$$

Additional constraints are imposed in Ref. 1 to generate a solution to the model following problem. K_x, in Eq. (2), assigns some of the closed-loop eigenvalues of the plant to the finite transmission zeroes of the plant, the finite transmission zeroes of the plant must be stable, a special matrix B^* (B^* is defined in Eq. 21 of Ref. 1) must be invertible and $\sigma_k \leq \sigma_{mk}$ for $k = 1,2,...l$ (σ_k and σ_{mk} are defined in Eqs. 5a and 6 of Ref. 1). In Kawahata's conclusion, it is indicated that a more general form of Eq. (3) will be investigated in future work where special compensators and derivatives of u_m will be included in Eq. (3).

The purpose of this correspondence is to point out that recently a more general solution to the model following problem has been solved in Refs. 2 and 3. We will briefly show that the model following result in Ref. 1 is a special case of the more general solution. In Ref. 2 it is not required that D and D_m be zero. The more general class of controllers allowed in Ref. 2 includes compensators and derivatives of u_m *if they are needed.* If simple state feedback is used as the regulation mechanism in the tracking control system it is not required that K_x assign some of the closed-loop eigenvalues to the finite transmission zeroes of the plant. It is possible to do so if desired, however. It is not required in Ref. 2 that the plant transmission zeroes be stable, provided the unstable transmission zeroes are common to the plant and model (Ref. 4). No relationship is required between σ_k and σ_{mk}, however, derivatives of u_m may become necessary in the tracking control law if $\sigma_k > \sigma_{mk}$. The solution in Ref. 2 is straightforward and is given in state space form. The solution discussed in Ref. 2 and more extensively detailed in Ref. 3 shows that regulation and tracking are two distinct problems and can be solved separately if desired. A summary of the results in Refs. 2 and 3 follows.

Perfect tracking of y_m by y occurs along the trajectories x^* and u^* which satisfy the following equation:

$$\begin{bmatrix} x^* \\ u^* \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} x_m \\ u_m \end{bmatrix} + \begin{bmatrix} \Omega_{11} \\ \Omega_{21} \end{bmatrix} \nu \tag{4}$$

$$\Omega_{11} \dot{\nu} = \nu - S_{12} \dot{u}_m \tag{5}$$

The feedforward matrices, S_{11}, S_{12}, S_{21}, and S_{22} satisfy Eq. (9) in Ref. 2, an algebraic equation. A solution to Eq. (9) in Ref. 2 requries that no transmission zero of the plant equal an eigenvalue of the model (see also Ref. 5). Kawahata does not discuss this requirement because the feedback gain in Ref. 1 performs a pole-transmission zero cancellation *before* solving the model-following problem. The other matrices in Eqs. (4) and (5), Ω_{11} and Ω_{21}, satisfy Eq. (14) in Ref. 2.

The x^* and u^* trajectories, by construction, satisfy the plant dynamics. If the error quantities, $\tilde{x} = (x - x^*)$, and $\tilde{u} = (u - u^*)$, are formed, as discussed in Ref. 3, they satisfy the equation

$$\dot{\tilde{x}} = F\tilde{x} + G\tilde{u} \tag{6}$$

i.e., the plant dynamics. At this point, any control law employing some form of state feedback to stabilize the tracking error can be designed for $\tilde{u}$ that a designer wishes. The particular type of feedback used in the model following control law derived by Kawahata is not explicitly discussed in Refs. 2 and 3 since the objective of Ref. 2 is to solve the model following problem. Model following and feedback control are distinct problems. Feedback control changes $\tilde{x}$ but does not alter x^*.

In order to obtain the control law form chosen by Kawahata [Eq. (3)], simple state feedback is combined with the model following solution

$$\tilde{u} = K_x \tilde{x} \tag{7}$$

$$\dot{\tilde{x}} = (F + GK_x)\tilde{x} \tag{8}$$

Substituting Eq. (4) into Eq. (7) and combining terms produces

$$u = K_x x + (S_{21} - K_x S_{11}) x_m + (S_{22} - K_x S_{12}) u_m + (\Omega_{21} - K_x \Omega_{11}) \nu \tag{9}$$

K_x in Eq. (9) can be chosen to place closed-loop eigenvalues of the plant to any desired set of symmetric stable eigenvalues. A linear quadratic regulator (LQR) can be used to obtain K_x, if desired, by weighting $\tilde{x}$ and $\tilde{u}$ in the LQR cost function (Ref. 3). Perfect tracking occurs asymptotically (i.e., $\tilde{x} \rightarrow 0$) if the plant and model initial conditions are arbitrary.

Received Jan. 22, 1981; revision received May 8, 1981.

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Table 1 Inverse and feedforward matrix solutions

$$S_{11} = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ -0.119 & 0.297 & -1.37\times10^{-8} & 0.812 & 0.0 \\ 0.00263 & 0.00444 & 0.388 & 0.0217 & -8.31\times10^{-8} \\ -0.00221 & -0.00931 & 0.0 & 1.02 & 0.0 \\ 13.4 & -23.5 & -3.75\times10^{-7} & 3.82 & 0.0128 \end{bmatrix}$$

$$S_{12} = 0$$

$$S_{21} = \begin{bmatrix} 0.00126 & -0.00371 & -0.103 & -0.0112 & 0.0 \\ 0.481 & -0.283 & -39.5 & -3.19 & 1.72\times10^{-4} \end{bmatrix}$$

$$S_{22} = \begin{bmatrix} 0.0186 & 1.062\times10^{-5} \\ 3.52\times10^{-9} & 0.319 \end{bmatrix}$$

$$\Omega_{11} = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & -0.7924712 & 0.0 & 42.476485 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & -0.0147849 & 0.0 & 0.7924712 & 0.0 \\ 340.13605 & -21.552343 & 0.0 & 1155.2063 & 0.0 \end{bmatrix}$$

$$\Omega_{21} = \begin{bmatrix} 0.0 & 0.00883 & -0.128 & -0.685 & 0.0 \\ 8.50 & -0.539 & 0.0 & 28.9 & 0.025 \end{bmatrix}$$

It can be shown, using results from Ref. 6, that Kawahata eliminates the last term in Eq. (9) by choosing some elements in K_x so that

$$(\Omega_{21} - K_x\Omega_{11})\nu = 0 \tag{10}$$

The minimal realization of the generalized state space compensator in Eq. (5) (the dynamical equation is also sometimes referred to as a descriptor system) cancels transmission zeroes of the plant not common between the plant and model. If some of the elements in K_x can be chosen to satisfy Eq. (10), then some of the closed-loop eigenvalues of the plant equal all of the *finite* plant transmission zeroes as discussed by Kawahata. Plant transmission zero locations do not always satisfy desirable handling qualities specifications.

Reworking the example in Ref. 1 using Eqs. (4) and (9) where the plant is a Beechcraft Model 65 and the model to be followed is that of a Boeing 747 yields the results in Table 1. Using Table 1, and K_x from Table 2 in Ref. 1, the values for $S_{21} - K_xS_{11}$ and $S_{22} - K_xS_{12}$ are in agreement with K_{xm} and K_{um} in Table 2 of Ref. 1 to three significant digits. Note that S_{12} is the null matrix, causing the minimal realization of ν to be $\nu = 0$. The last term in Eq. (9) is zero for any value of K_x, a feature of this example that is also pointed out by Kawahata.

References

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[2]Broussard, J.R. and O'Brien, M.J., "Feedforward Control to Track the Output of a Forced Model," *IEEE Transactions on Automatic Control*, Vol. AC-25, Aug. 1980, pp. 851-854.

[3]O'Brien, M.J. and Broussard, J.R., "Feedforward Control to Track the Output of a Forced Model," *Proceedings of the 17th IEEE Conference on Decision and Control*, San Diego, Calif., Jan. 1979, pp. 1149-1155 (a longer version of Ref. 2).

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[6]Mabius, L.E. and Kalnitsky, K.C., "Model Reference Adaptive Control Theory for Power Plant Control Applications," The Analytic Sciences Corporation, Rept. No. TR-1489-1, June 1980.

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Reply by Author to J.R. Broussard and L.E. Mabius

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I WISH to thank Mr. Broussard and Mr. Mabius for their Comment on my work[1] and for drawing attention to their Ref. 2. When my manuscript was prepared, Ref. 2 was not yet available. Even so, the two papers complement each other.

In Ref. 1, a practically oriented design technique was stressed rather than a more theoretically oriented approach to the model following system. The appearance of time

Received May 14, 1981.
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